

## AN ANALYTICAL STUDY ON BUCKLING BEHAVIOR OF CNT/POLYMER COMPOSITE PLATES USING THE FIRST ORDER SHEAR DEFORMATION THEORY

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Most plates used in engineering structures such as aircraft wings, ship ducts and buildings, although quite capable of resisting tensile loads, are poor in withstanding compression. In order to avoid premature failure under compression, it is important to know buckling behavior of the plate. This article primarily deals with the analytical study of buckling behavior of a carbon nanotube reinforcing polymer composite plates based on the first order shear deformation theory by employing Mori-Tanaka micromechanics approach to obtain elastic properties. In this investigation, an attempt is made for evaluating the effect of plate thickness, CNT volume fraction, stacking sequence and CNT radii on the buckling of plates.

*Keywords:* buckling, carbon nanotube, composite plates, micromechanics, FSDT

### 1. Introduction

Since the recognized discovery of Carbon Nanotubes (CNTs), Iijima (1991), they have attracted an intense interest among researchers in various disciplines due to their stupendous mechanical, electrical and thermal properties (Saito *et al.*, 1999; Ounaies *et al.*, 2003; Weisenberger *et al.*, 2003). The remarkable electrical properties have made CNTs excellent candidates to act as reinforcement in a wide range of applications such as nanosensors and atomic transportation (Arash and Wang, 2013; Wang, 2008). In addition, the outstanding physical and mechanical properties of CNTs, for example an ultra-high Young's modulus of the order of 1 TPa and tensile strength of 200 GPa (Lau and Hui, 2002; Demczyk *et al.*, 2002), stimulated the interest in using CNTs as filler materials in polymer composites to make ultra light structural materials with enhanced mechanical properties.

In the past decade, the wide-ranging experimental and theoretical studies conducted on carbon nanotubes have focused on mechanical characterization and modeling aspects of reinforced polymer composites to assist the development of nanocomposites (Xiao *et al.*, 2005; Zhang Z. *et al.*, 2010; Zhang Y. *et al.*, 2013; Arash *et al.*, 2014; Silani *et al.*, 2014; Gates *et al.*, 2005; Tserpes and Papnikos, 2005). Even though these studies are fairly useful in estimating properties of the nanomaterials, their use in actual structural applications is the ultimate idea behind the development of this highly sophisticated class of materials. However, the literature shows that much of work in this direction is not accomplished. As such, there is a need to examine the macro behavior of the material in actual structural elements such as beams and plates. The behavior of beams under a static loading was studied by Wuite and Adali (2005) for different CNT volume of fractions and nanotube diameters. Based on different assumptions for displacement fields, different theories for plate analysis have been devised. These theories can be divided into three major categories, individual layer theories, equivalent single-layer (ESL) theories, and

three-dimensional elasticity solution procedures. These categories are further divided into sub-theories by the introduction of different assumptions. For example, the second category includes the classical laminate plate theory (CLPT), the first-order and higher-order shear deformation theories (FSDT and HSDT). Analytical observations on static and dynamic behavior of plates by using CLPT were made by Madhu *et al.* (2012).

For the implementation of Carbon nanotube reinforced polymer (CNRP) composites in structural applications, accurate property-microstructure relations are required in the form of micromechanics models. In the present investigation, micromechanics properties of CNRP are computed using Mori-Tanaka method as given in (Popov *et al.*, 2000; Shi *et al.*, 2004; Wuite and Adali, 2005; Madhu *et al.*, 2012). The effects of the characteristics developed in these models are investigated on the buckling behavior of CNRP composite plates using FSDT with a view towards assessing the effectiveness of these materials in the design of structural nanocomposites.

## 2. Micro-mechanics model

The micro-mechanical model involves an elastic and isotropic polymer reinforced with straight, aligned and infinitely long single walled carbon nanotubes (SWCNT). The plate under the study is composed of polystyrene as the matrix with the Young modulus and Poisson's ratio of  $E_m = 1.9$  GPa and  $\nu_m = 0.3$ , respectively. Each SWCNT is considered to be solid and transversely isotropic, and their Hill's elastic constants are taken from Popov *et al.* (2000). The SWCNT radius is assumed to be  $10 \text{ \AA}$  for all the cases, otherwise mentioned, for which the representative values of the elastic constants are:  $n_r = 450$  GPa,  $k_r = 30$  GPa,  $m_r = p_r = 1$  GPa and  $l_r = 10$  GPa. The bonding at the nanotube-polymer interface is taken to be perfect. The resulting composite is also considered as transversely isotropic and its constitutive stress strain relations  $\sigma = C\varepsilon$  can be expressed as follows

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} n & l & l & 0 & 0 & 0 \\ l & k+m & k-m & 0 & 0 & 0 \\ l & k-m & k+m & 0 & 0 & 0 \\ 0 & 0 & 0 & 2m & 0 & 0 \\ 0 & 0 & 0 & 0 & 2p & 0 \\ 0 & 0 & 0 & 0 & 0 & 2p \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{Bmatrix} \quad (2.1)$$

where  $k$  is the plane-strain bulk modulus normal to the fiber direction,  $n$  is the uni-axial tension modulus in the fiber direction (1),  $l$  is the associated cross modulus,  $m$  and  $p$  are the shear moduli in planes normal and parallel to the fiber direction, respectively, and they are Hill's elastic constants. A composite with a reinforcing phase volume fraction  $c_r$ , matrix Young's modulus  $E_m$  and matrix Poisson's ratio  $\nu_m$  is considered. Using the Mori-Tanaka method, Hill's elastic moduli are found to be (Popov *et al.*, 2000; Shi *et al.*, 2004; Wuite and Adali, 2005; Madhu *et al.*, 2012)

$$\begin{aligned} k &= \frac{E_m \{ E_m c_m + 2k_r(1 + \nu_m)[1 + c_r(1 - 2\nu_m)] \}}{2(1 + \nu_m)[E_m(1 + c_r - 2\nu_m) + 2c_m k_r(1 - \nu_m - 2\nu_m^2)]} \\ l &= \frac{E_m \{ c_m \nu_m [E_m + 2k_r(1 + \nu_m)] + 2c_r l_r(1 - \nu_m^2) \}}{(1 + \nu_m)[2c_m k_r(1 - \nu_m - 2\nu_m^2) + E_m(1 + c_r - 2\nu_m)]} \\ n &= \frac{E_m^2 c_m(1 + c_r - c_m \nu_m) + 2c_m c_r(k_r n_r - l_r^2)(1 + \nu_m)^2(1 - 2\nu_m)}{(1 + \nu_m)[2c_m k_r(1 - \nu_m - 2\nu_m^2) + E_m(1 + c_r - 2\nu_m)]} \\ &+ \frac{E_m [2c_m^2 k_r(1 - \nu_m) + c_r n_r(1 - 2\nu_m + c_r) + 4c_m c_r l_r \nu_m]}{2c_m k_r(1 - \nu_m - 2\nu_m^2) + E_m(1 + c_r - 2\nu_m)} \end{aligned} \quad (2.2)$$

$$p = \frac{E_m[E_m c_m + 2(1 + c_r)p_r(1 + \nu_m)]}{2(1 + \nu_m)[E_m(1 + c_r) + 2c_m p_r(1 + \nu_m)]}$$

$$m = \frac{E_m[E_m c_m + 2m_r(1 + \nu_m)(3 + c_r - 4\nu_m)]}{2(1 + \nu_m)\{E_m[c_m + 4c_r(1 - \nu_m)] + 2c_r m_r(3 - \nu_m - 4\nu_m^2)\}}$$

where  $k_r$ ,  $l_r$ ,  $m_r$ ,  $n_r$  and  $p_r$  are Hill's elastic moduli for the reinforcing phase. The expressions for the moduli of the CNTRC as functions of the stiffness constants are determined for a unidirectional composite as follows

$$E_1 = n - \frac{l^2}{k} \quad E_2 = \frac{4m(kn - l^2)}{kn - l^2 + mn} \quad G_{12} = 2p \quad \nu_{12} = \frac{l}{2k} \quad (2.3)$$

### 3. Basic equations

The buckling behavior of a symmetric laminated carbon nanotube reinforced polymer square plate is to be studied under the first order shear deformation theory (FSDT), and the basic equations are summarized in this section.

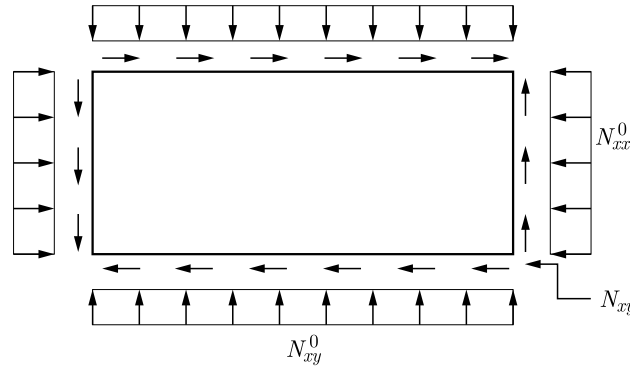


Fig. 1. Buckling of a plate under in-plane compressive edge forces

For laminated composite plates subjected to only in-plane loads, the constitutive relations for the buckling under FSDT are taken from Reddy (2004). A parametric study is carried out to find the minimum buckling load for laminated composite plates which occurs at  $m = n = 1$ . In the buckling analysis, it is assumed that the loads are in-plane forces only.

For simplicity

$$\hat{N}_{xx} = -N_0 \quad \hat{N}_{yy} = -kN_0 \quad k = \frac{\hat{N}_{yy}}{\hat{N}_{xx}} \quad (3.1)$$

Therefore:

— for uniaxial compression  $k = 0$

$$\hat{N}_{xx} = -N_0 \quad \hat{N}_{yy} = 0 \quad (3.2)$$

— for biaxial compression  $k = 1$

$$\hat{N}_{xx} = -N_0 \quad \hat{N}_{yy} = -N_0 \quad (3.3)$$

The critical buckling load is nondimensionalised by using

$$\bar{N} = N_{cr} \frac{a^2}{E_2 h^2} \quad (3.4)$$

here  $\bar{N}$  is the nondimensionalised critical buckling load,  $N_{cr}$  is the critical buckling load,  $a$  is length of the plate,  $E_2$  is Young's modulus in the transverse direction and  $h$  is thickness of the laminated composite plate.

#### 4. Buckling analysis

A proficient MATLAB code is developed for the buckling analysis using FSDT of carbon nanotube reinforced polymer (CNRP) composite plates. A twofold validation is done to the code, one for validating the elastic properties of CNRP and the other for validating the critical buckling load of composite plates under FSDT. First, the elastic properties of CNRP obtained from Mori-Tanaka micromechanics model are validated with the published results (Wuite and Adali, 2005) and presented in Fig. 2.

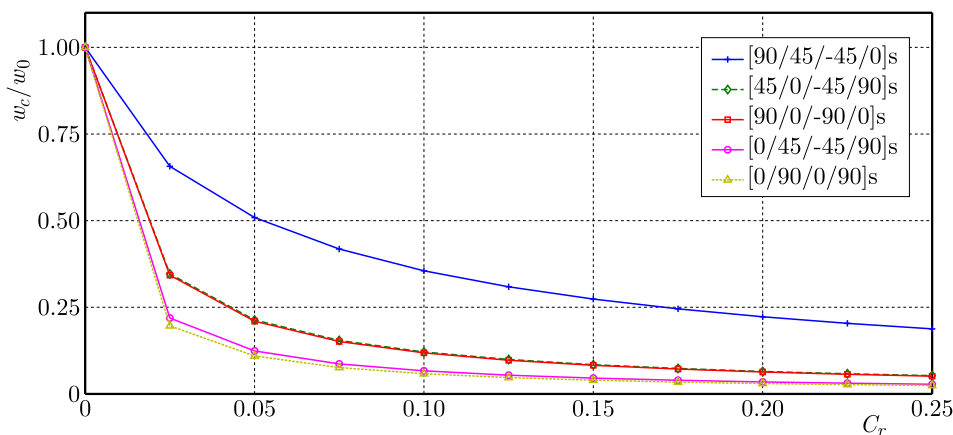


Fig. 2. Curves of the maximum deflection plotted against the fiber volume fraction for various stacking sequences for a simply supported beam subjected to CPL

Further to validate the buckling with the published results, the effect of shear deformation on the nondimensionalised critical buckling loads  $\bar{N}$  of symmetric laminated plates with elastic properties  $E_1 = 25E_2$ ,  $G_{12} = G_{13} = 0.5E_2$ ,  $G_{23} = 0.2E_2$ ,  $\nu_{12} = 0.25$ ,  $K = 5/6$  is calculated for uniaxial as well as biaxial compression and presented in Table 1 along with the corresponding published results (Reddy, 2004).

**Table 1.** Comparison of the nondimensionalised critical buckling load with the published results

$a/h$	[0/90/0]		[0/90/0/90/0]	
	$\bar{N}$	Reddy (2004)	$\bar{N}$	Reddy (2004)
Uniaxial compression				
10	15.248	15.289	16.254	16.309
20	20.602	20.628	21.094	21.125
25	21.552	21.568	21.893	21.917
50	22.971	22.978	23.071	23.078
100	23.361	23.363	23.384	23.389
Biaxial compression				
10	7.569	7.644	8.070	8.154
20	10.293	10.314	10.522	10.562
25	10.771	10.784	10.930	10.958
50	11.486	11.489	11.531	11.539
100	11.681	11.682	11.692	11.695

The results obtained are found to be in excellent agreement with the published results. Having validated the composite plate results, the MATLAB code is extended to the CNRP

composite plate to make an attempt to analyze the buckling behavior of these plates under FSDT.

An eight-layer simply supported symmetric square CNRP composite plate is subjected to both uniaxial and biaxial compression to study the buckling behavior. For the CNRP composite plate, the critical buckling load occurs at  $m = n = 1$ , and the variation in critical buckling load with respect to side-to-thickness ratio  $a/h$  of the plate and CNT volume fraction (fiber volume fraction) for various stacking sequences is shown in Figs. 3 and 4.

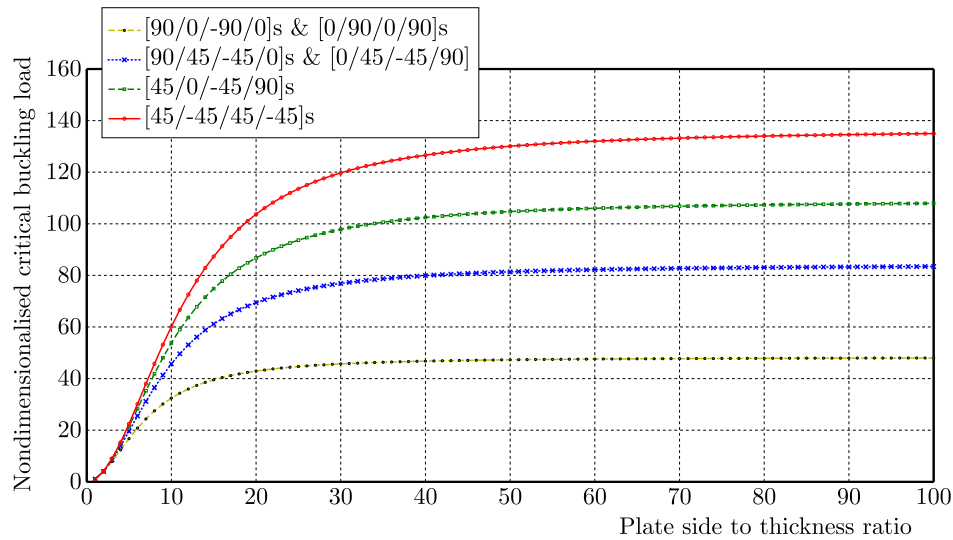


Fig. 3. Curves of the nondimensionalised critical buckling load plotted against the plate side-to-thickness ratio for various stacking sequences for a simply supported plate subjected to uniaxial compression

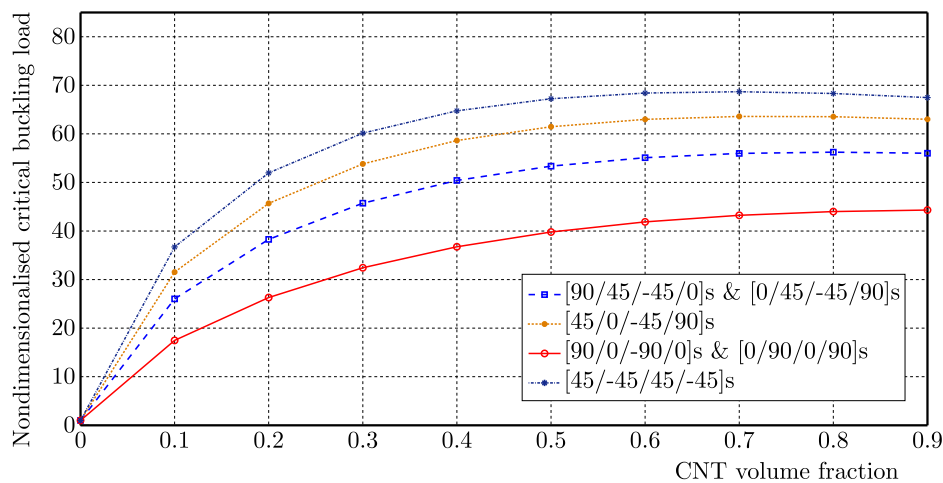


Fig. 4. Curves of the nondimensionalised critical buckling load plotted against the CNT volume fraction for various stacking sequences for a simply supported plate subjected to uniaxial compression

From the above results, it is observed that the angle-ply stacking sequence  $[45/-45/45/-45]$  s exhibits more resistance to buckling than the cross-ply laminating schemes  $[0/90/0/90]$  s and  $[90/0/-90/0]$  s for the same percentage of CNT reinforcement. For all the stacking sequences, the effect of shear deformation on the critical buckling load is clearly observed in thick plates ( $0 < a/h \leq 20$ ), and it is insignificant in the case of thin plates ( $a/h > 20$ ) (Reddy, 2004). However, the effect of shear deformation across the thickness on the critical buckling load is the same even the plate is subjected to biaxial compression ( $k = 1$ ) with actual values lesser than those of uniaxial compression ( $k = 0$ ). The comparison between the nondimensionalised critical

buckling loads of uniaxial and biaxial compression for the stacking sequence  $[90/45/-45/0]_s$  at CNT volume fraction 0.3 is shown in Fig. 5.

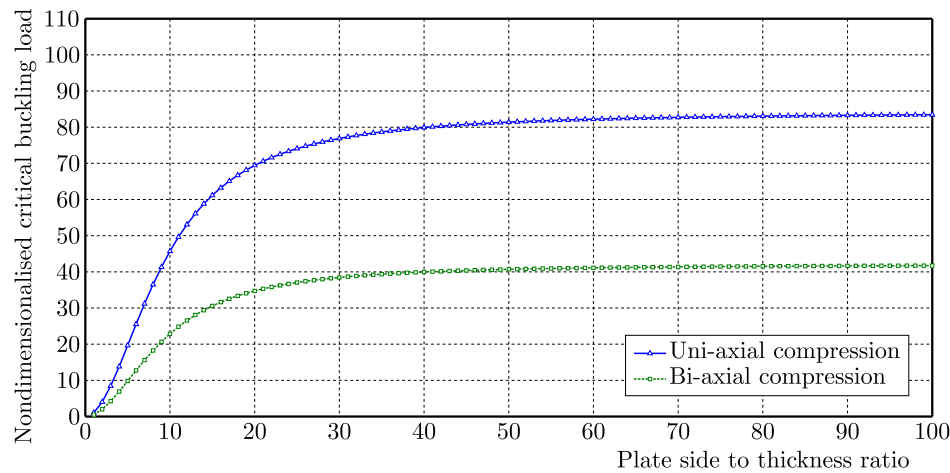


Fig. 5. Curves of the nondimensionalised critical buckling load plotted against the plate side-to-thickness ratio for various loads and stacking sequences  $[90/45/-45/0]_s$  for a simply supported plate

It is knowledgeable that the nondimensionalised critical buckling load in biaxial compression is about 0.5 times lesser than that of uniaxial compression for various stacking sequences, CNT radii and for various fiber volume fractions. So the results are presented only for uniaxial compression hereafter.

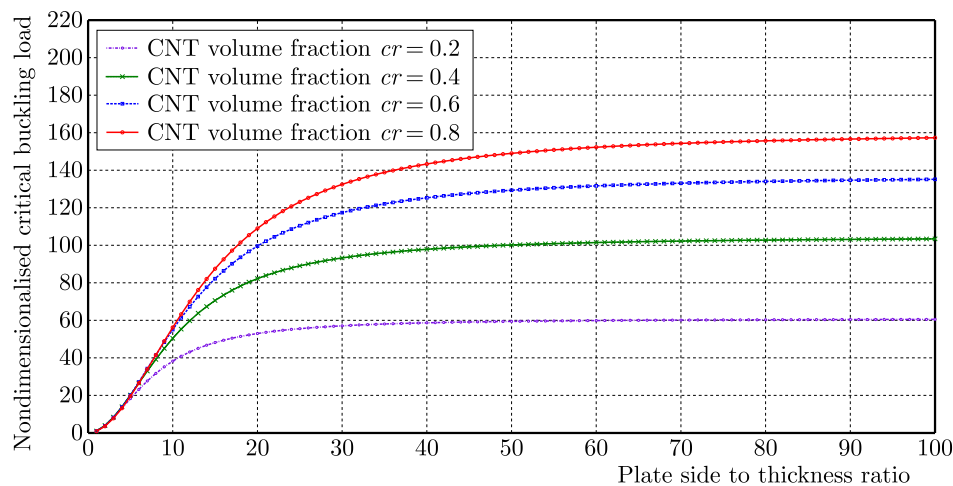


Fig. 6. Curves of the nondimensionalised critical buckling load plotted against the plate side-to-thickness ratio for fiber volume fractions and stacking sequences  $[90/45/-45/0]_s$  for a simply supported plate subjected uniaxial compression

The effect of fiber volume fraction on the critical buckling load is presented in Fig. 6. It is obvious that the critical buckling load increases as the fiber volume fraction in the composite increases, as CNTs are stiffer and stringer. It is further investigated to discover the effect of CNT radius on the critical buckling load, and it is observed from the following graph (Fig. 7) that for the same stacking sequence, the effect of nanotube radius on the critical buckling load is insignificant. The same is observed for various stacking sequences, and the results are shown in Fig. 8.

From the above results, it is noticeable that a higher stiffness can be obtained by reinforcing CNTs with smaller diameters and, thereby, a decrease in the critical buckling load can also be achieved.

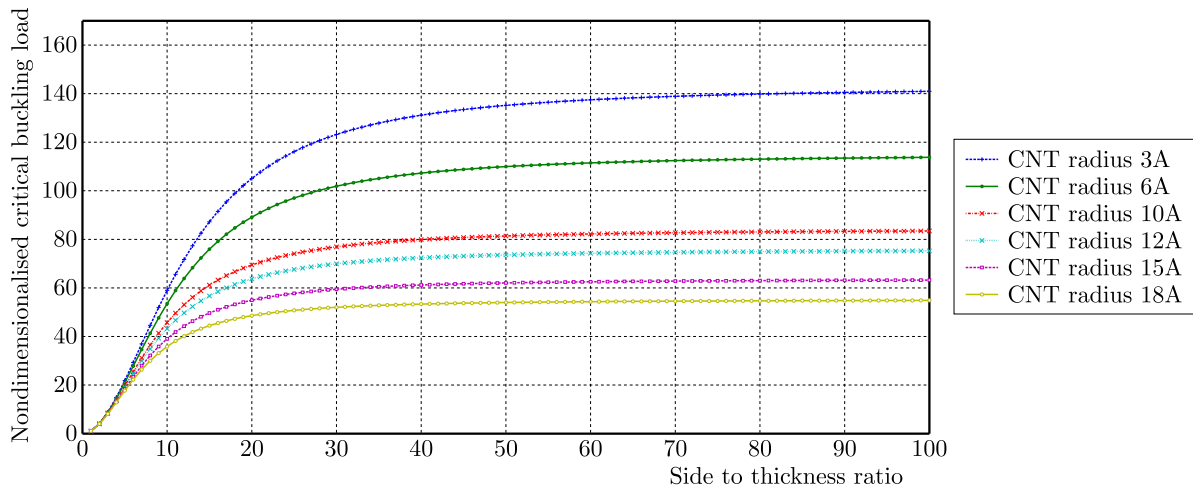


Fig. 7. Curves of the nondimensionalised critical buckling load plotted against the plate side-to-thickness ratio for various CNT radii and stacking sequences  $[90/45/-45/0]_s$  for a simply supported plate subjected uniaxial compression

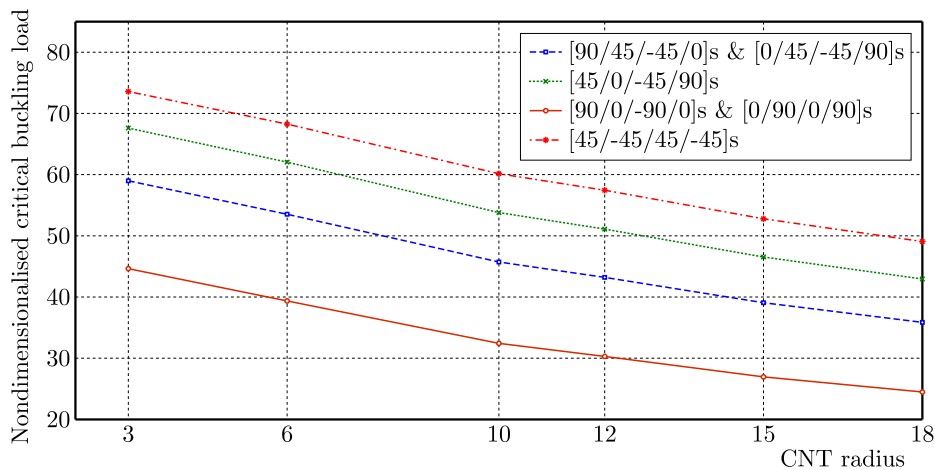


Fig. 8. Curves of the nondimensionalised critical buckling load plotted against CNT radii for various stacking sequences for a simply supported plate subjected uniaxial compression ( $k = 0$ )

## 5. Conclusions

The buckling behavior of a CNT reinforced polymer composite plate is studied by using a MATLAB code. The Mori-Tanaka micromechanics model is adopted for determining the elastic constants of the CNT reinforced polymer composite material in terms of nanotube volume fractions. It is shown that they are in good agreement with the published results. The first order shear deformation theory (FSDT) is considered for determination of the critical buckling loads of plates.

A twofold validation is done to the code; one is to validate the elastic constants of the CNRP composite and the second to validate the FSDT. In the case of CNRP composite plates under FSDT, it is found that the effect of side to thickness ratio on the critical buckling load of thick plates is phenomenal. The results markedly show the diminishing effect of transverse shear deformation on the critical buckling load of thick plates, the effect being negligible for side to thickness ratios greater than 20 i.e., for thin plates. It is also investigated that the stacking sequence of the laminated plate is yet another important parameter for determination of the buckling behavior of the plates. It is observed that the buckling resistance of the angle-ply

laminating scheme is less than that of other stacking sequences considered in the study. The influence of CNT radii is also examined on the buckling of the plates, and it is proposed to use a smaller radius to obtain a higher buckling resistance. The effect of the volume fraction of the CNT reinforcements on the buckling of the plate is studied, and it is found that the increase in the fiber volume fraction decreases the critical buckling load.

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